CONTENT

VIRTUAL INVERSE RHEOMETRY

- Power-law description : $\eta(\mathbf{u}) = \eta_0(T)\|\underline{D}(\mathbf{u})\|_F^{s-2}$

➡ uncertain parameters:

Exponent s

Consistency η_0

Temperature T(z): linear decrease from bottom to surface

- « Ill-posed »: $\eta(\mathbf{u}) \longrightarrow +\infty$ when $\|\underline{D}(\mathbf{u})\| \to 0$ since $s-2<0$

➡ singularity at free surface (vanishing shear)

Goal:

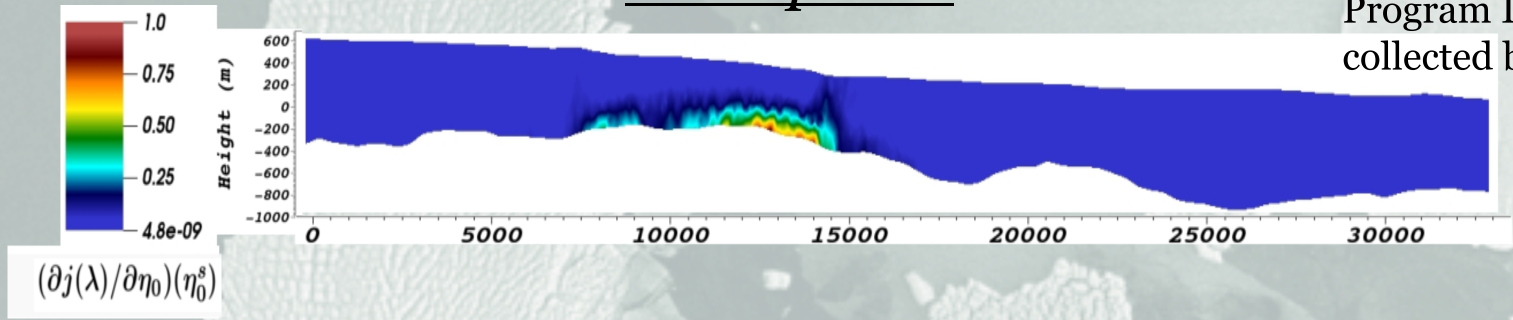
➡ Determine the importance of rheological parameters

➡ Provide calibration/identification

Fluid consistency $\eta_0(T)$: sensitivity

No-slip case

Program ICECAP, 2010, collected by B. Legrésy



- High sensitivity = highly sheared area
  - ➡ localized at the accelerating zone (bump)
  - ➡ concentrated at bottom (no-slip)

Behaviour at high shear-rate dominant

- Sensitivity independant of the vertical temperature gradient

Sliding case

- Relaxation of the high shearing at bottom ➡ distribution of the shear-rate

Vertical temperature gradient ↗ ➡ Velocity gradient (shearing) ↗ ➡ Sensitivity localized at the bottom

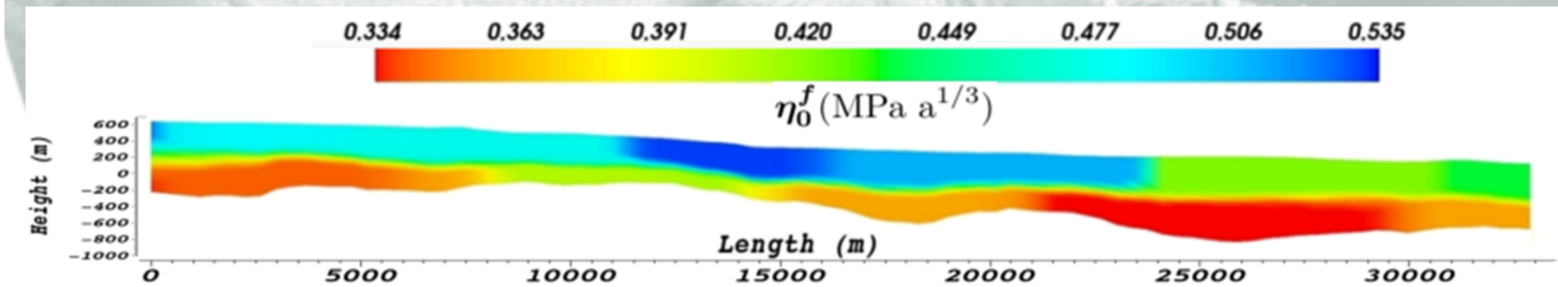
Fluid consistency $\eta_0(T)$: identification

- $\eta_0(T)$: spatially distributed value (to be inferred)
  → requires regularization (Tykhonov)
- Absolute sensitivity: low compared to the exponent one
- Cost function :

$$j(\eta_0) = \int_{\Gamma_s} \|u_s^{obs} - u_s(\eta_0)\|_2^2 \; d\mathbf{x} + \gamma_1 \int_{\Gamma_s} \|\partial_x \eta_0\|_2^2 \, ds + \gamma_2 \int_{\Gamma_s} \|\partial_z \eta_0\|_2^2 \, ds$$

- Regularization: *good a priori required* on the temperature gradient

Given exact ratio: $\gamma_1 = \gamma_2 / 7$, Initial-guess : surface value



Synthetic surface data: 0.1% noise
Final error ~ 9%

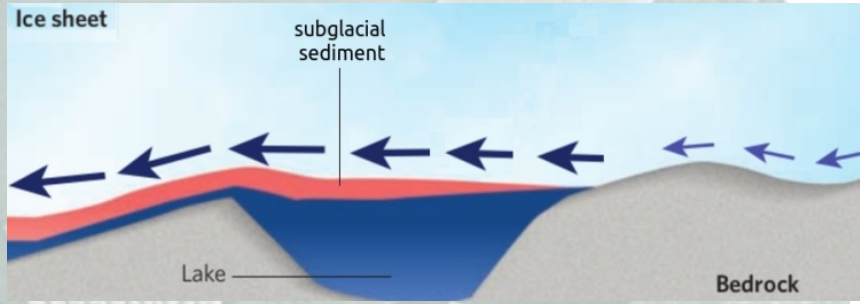
Otherwise ill-posed inverse problem, converge to wrong optimal value

Results presented in *Martin-Monnier, J. Geophysical Research, submitted*

CONTENT

THE BASAL SLIPPERINESS

- Major component of ice dynamics
  - *models heterogeneous interfacial layer*



Regelation, enhanced creep, subglacial hydrology, basal sediment deformation

  - *multiscale (spatial & temporal)*

$$|\tau_b|^{m-1}\tau_b = \boldsymbol{\beta} u_b^{\tau}$$

1 parameter involving various processes

- Generally unmeasurable

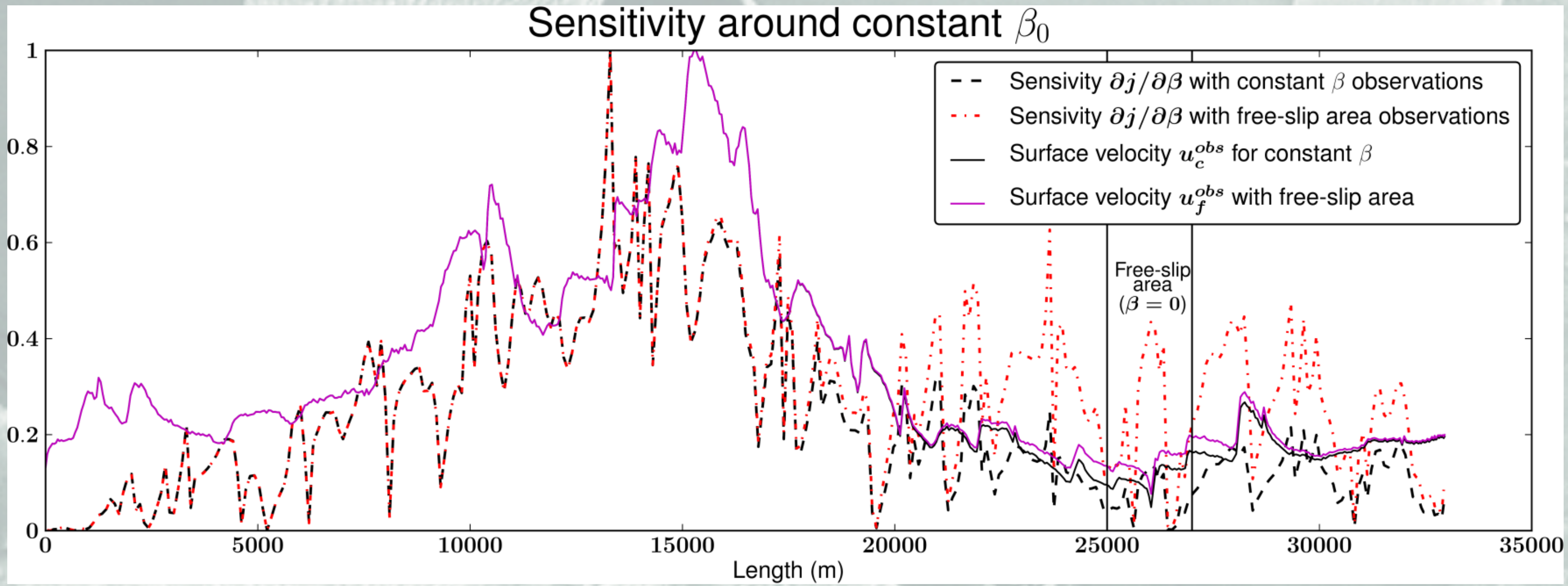
Goal:

➡ Sensitivity analysis: understand β *modelling role*

➡ Identification: infer the *bedrock-ice interaction* properties

Sensitivity to a free-slip area

Experiment 1: *effect of a local free-slip on the sensitivity to friction*



- Sensitivity correlated to the surface velocities (hence the topography)
- Local free-slip area (2km) ➡ large high sensitivity area (~10km)
  ➡ almost no effect on u_s
- Transmission of basal movement to the surface: **filtered** and **non-local**

Sensitivity to a free-slip area

- *Quasi-static sensitivities:*

  given a snapshot (surface & bedrock topography + surface velocities)

  → help to see beyond filtering and non-local

- *Free-slip area:*

  possibly local in space and time (e.g. fluctuation of subglacial meltwater pressure)

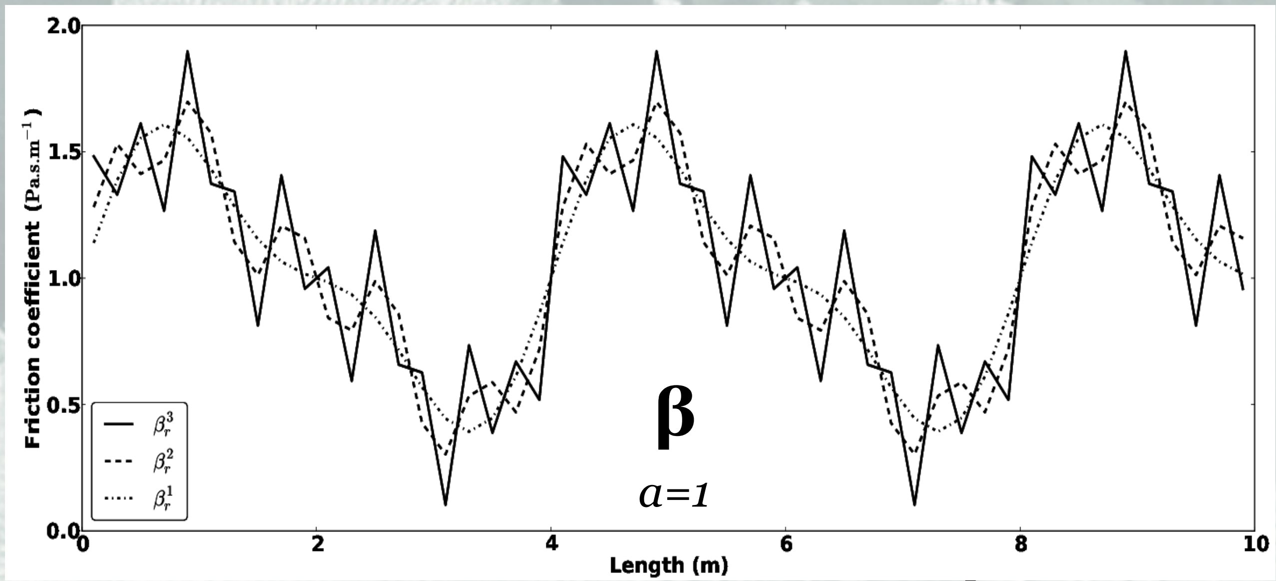
↓

Sensitivity analysis could allow to *detect momentary increase* in sliding rate (e.g. « *stick-slip* » phenomena)

Sensitivity and slip ratio r

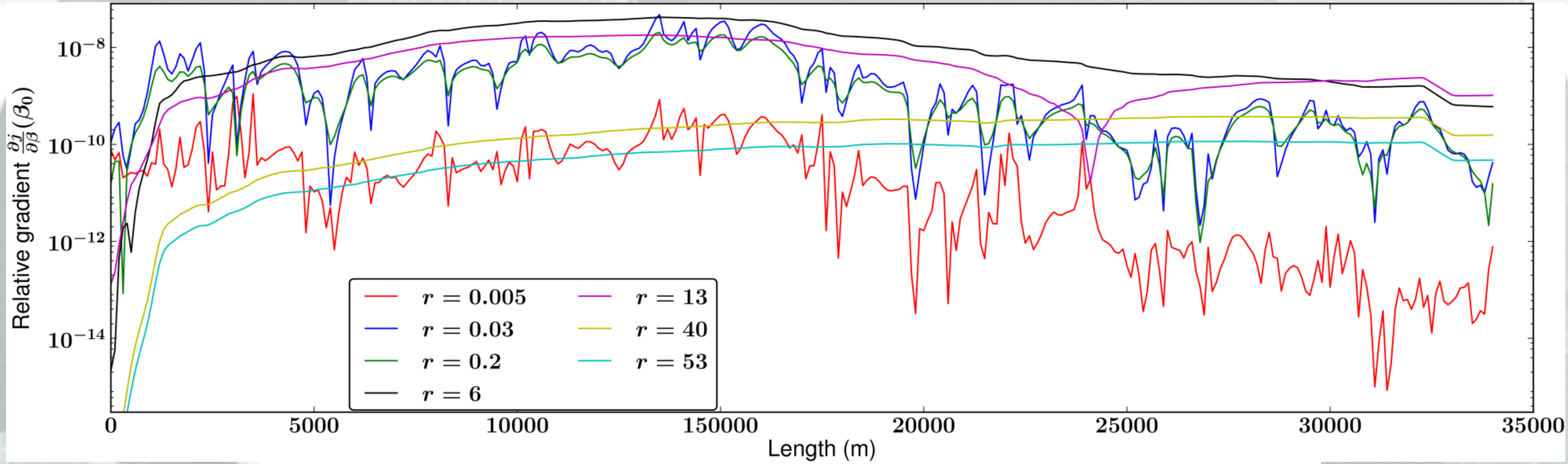
Experiment 2: effect of the slip ratio on the sensitivity to friction

- Slip ratio $r = \overline{u_b}/|\overline{u_s} - \overline{u_b}|$: balance between deformation and sliding
- Various frequencies in the friction coefficient β



$$\beta_r^n(x) = a + \frac{a}{2}\sin\left(\frac{2\pi x}{\lambda}\right) + \frac{a}{5}\sum_{i=1}^{n} f_i(x), \quad \beta = \beta_r^3$$

Sensitivity and slip ratio r



Sensitivity to the friction coefficient β for various slip ratio r

- *Low slip ratio*: r<0.01:
  high frequencies captured
  low absolute values
  ➡ filtering effects

- *High slip ratio*: r>5
  low frequency information
  average behaviour only
  ➡ non-local effects

Best situations to perform identification: *intermediate sliding*

Formulation of an incomplete adjoint

Recall: *adjoint model*

algorithmic differentiation of a fixed point type *iterative routine* (Stokes power-law): $y = \Phi(y, u)$

Reverse Accumulation (*Griewank et al.*, 1993)

1) derivation of the *computational graph* (node by node, one node n_i per function value Φ *)*

→ *one adjoint quantity per node* = $\dfrac{\partial \Phi}{\partial n_i}$

2) compute the adjoint state in reverse order

→ *adjoint state* = $\displaystyle\sum_i \frac{\partial \Phi}{\partial n_i}$ (chain formula)

Corollary: Adjoint state precision = Direct state precision

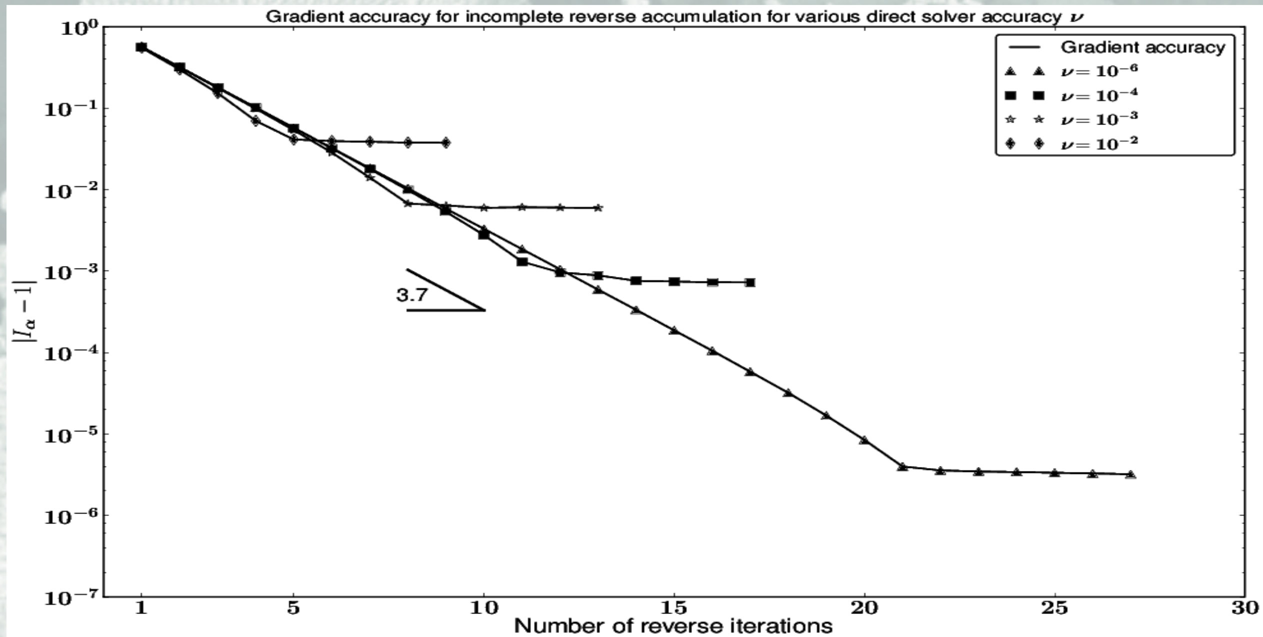
Christianson, 1992 **(~fixed point threshold)**

Formulation of an incomplete adjoint

Idea: retain « some » of the $\sum_i \dfrac{\partial \Phi}{\partial n_i}$ to compute the adjoint state

from 1 (« self-adjoint ») to all (complete adjoint)

What is the precision of the resulting *incomplete adjoint*?



➡ Adjoint precision adjustable a priori from data accuracy & direct solver precision

Friction coefficient invertibility

Experiment 3: Assess the limits of identifiability of the friction coefficient in Fourier space

- level of captured frequencies (in terms of thickness h)
- reconstructed amplitude

Setup:

- Synthetic data: 1% random noise
- Density: 1km (~1 ice thickness h) between each data point
- Realistic topography
- Non-linear friction law: $|\tau_b|^{m-1}\tau_b = \beta u_b^{\tau}$ with m=3

→ *Comparison of exact adjoint and « self-adjoint » method*

Friction coefficient invertibility

Slip ratio: r=0.005 (strong friction)

- *Complete adjoint:*

  High frequencies captured

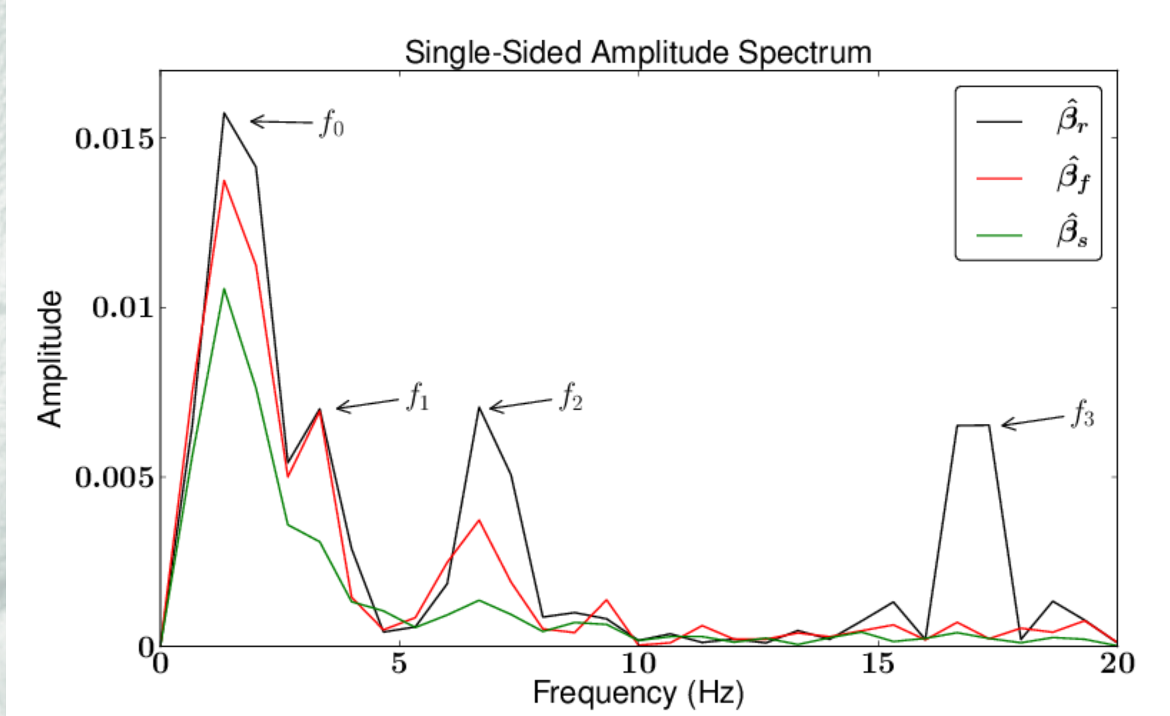
  Smaller amplitude

  Filtering dominant

- *« Self-adjoint »:*

  carrier frequency only

  ➡ not robust at low slip ratio



	f_0	f_1	f_2	f_3
Wavelengths w.r.t $h = 1$km	$20h$	$10h$	$5h$	$2h$
Wavelengths w.r.t $dx = 100$m	$200dx$	$100dx$	$50dx$	$20dx$
Wavelengths w.r.t $L = 33.3$km	1.66Hz	3.33Hz	6.66Hz	16.6Hz

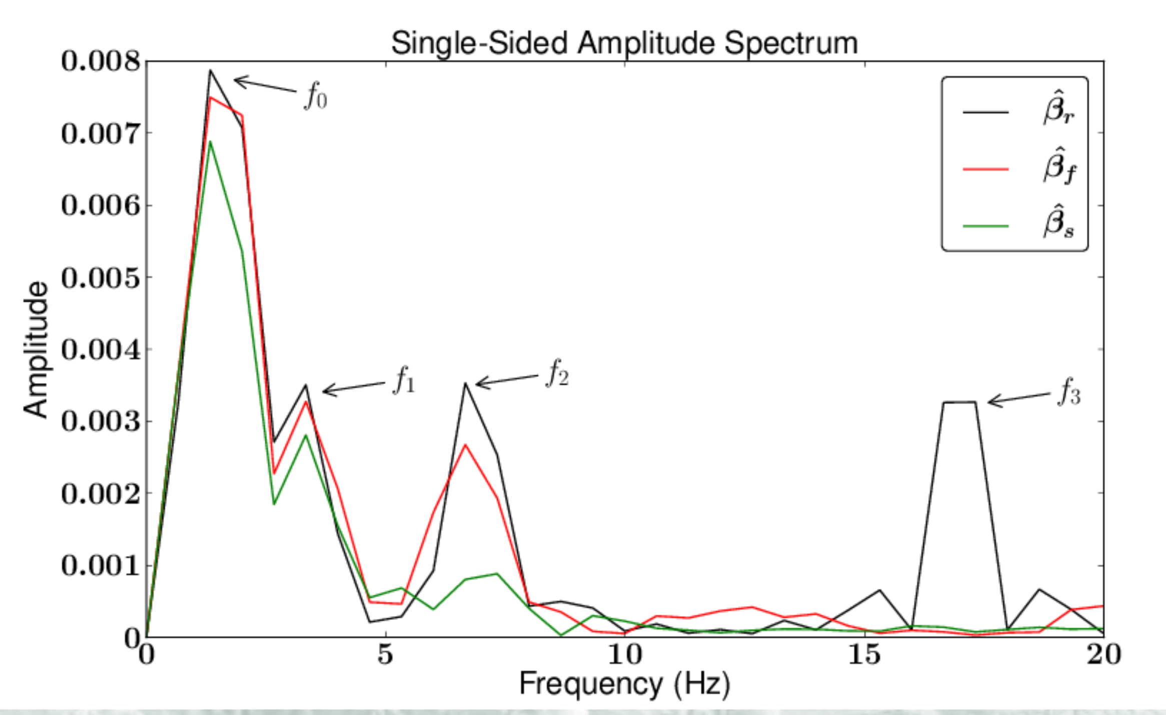
Friction coefficient invertibility

Slip ratio: r=0.5 (intermediate sliding)

- *Complete adjoint:*
  accurate up to
  5h wavelengths

- *« Self-adjoint »:*
  accurate up top
  10h wavelengths

Complete adjoint
more accurate



	f_0	f_1	f_2	f_3
Wavelengths w.r.t $h = 1$km	$20h$	$10h$	$5h$	$2h$
Wavelengths w.r.t $dx = 100$m	$200dx$	$100dx$	$50dx$	$20dx$
Wavelengths w.r.t $L = 33.3$km	1.66Hz	3.33Hz	6.66Hz	16.6Hz

Friction coefficient invertibility

Slip ratio: r=50 (rapid sliding)

- *Complete adjoint:*

Amplitude well represented

High frequencies not captured

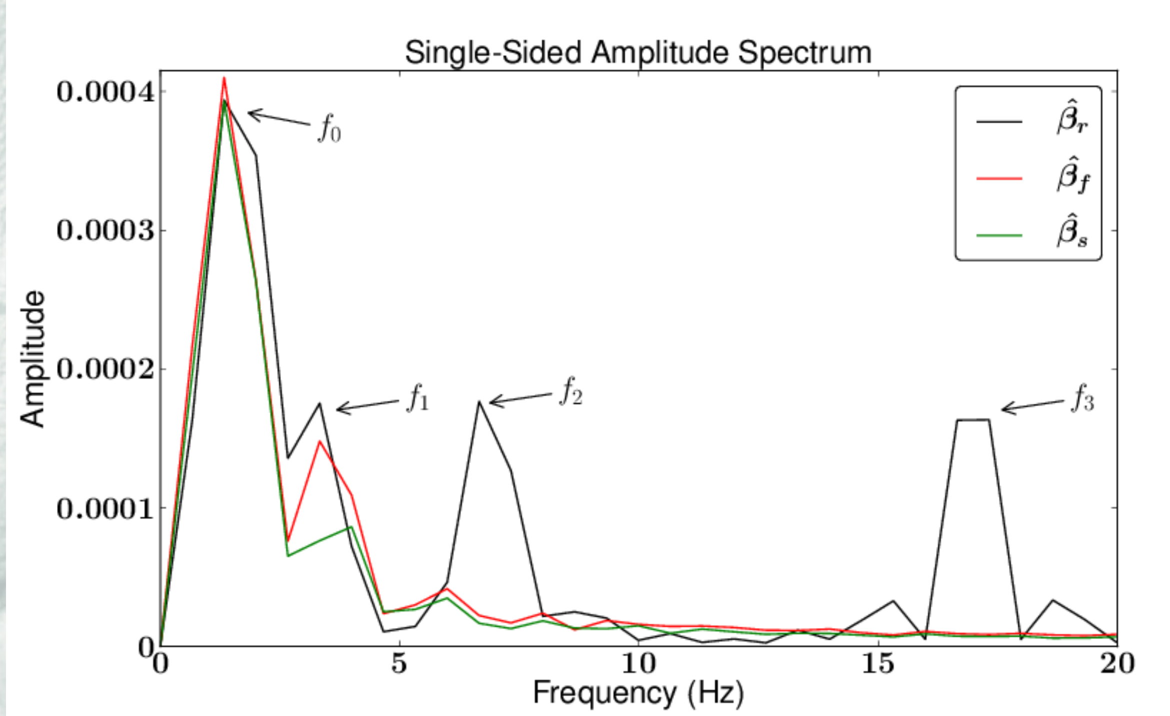
Non-local effects dominant

accurate up to 10h wavelengths

- *« Self-adjoint »:*

Only the carrier wave

accurate up to 20h wavelengths



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Results presented in *Martin-Monnier, The Cryosphere, submitted*

Overview

3) Basal sliding

- high frequency events captured (~2x thickness)
- detection of momentary basal slipperiness variation (« stick-slip »)

Martin-Monnier, SIAM-SISC, submitted

- Adjustable incomplete adjoint (from data accuracy & direct solver precision)

- Assessment of transmission of basal variability:

Friction	**Strong**	**Intermediate**	**Low**
Slip ratio	Small	~unity	Higher
Sensitivity	low	high	smooth
Transmission of basal variability	filtered ~5h (approx. amplitude)	« optimal » ~5h	non-local ~10h

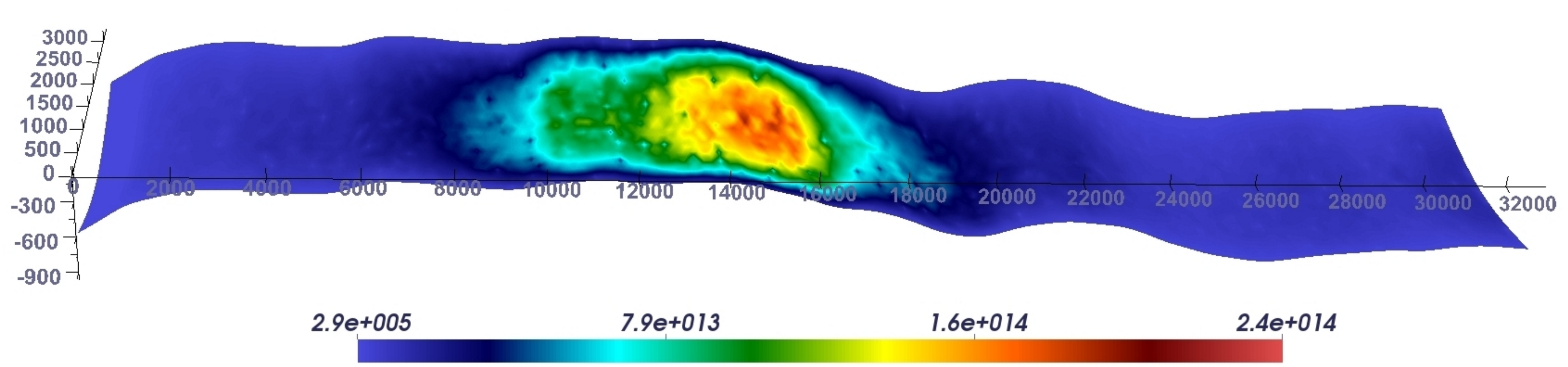
Martin-Monnier, The Cryosphere, submitted

Some perspectives

DassFlow-3D operational: tetrahedra, velocity-pressure formulation

(mainly developed by R. Madec – Post-doc IMT, ANR AMAC & ADAGe 2009-2013)

Available online open-source soon



Sensitivity with respect to the basal slipperiness

Madec-Martin-Monnier, under preparation

Some perspectives

DassFlow-3D operational: tetrahedra, velocity-pressure formulation
(mainly developed by R. Madec – Post-doc IMT, ANR AMAC & ADAGe 2009-2013)

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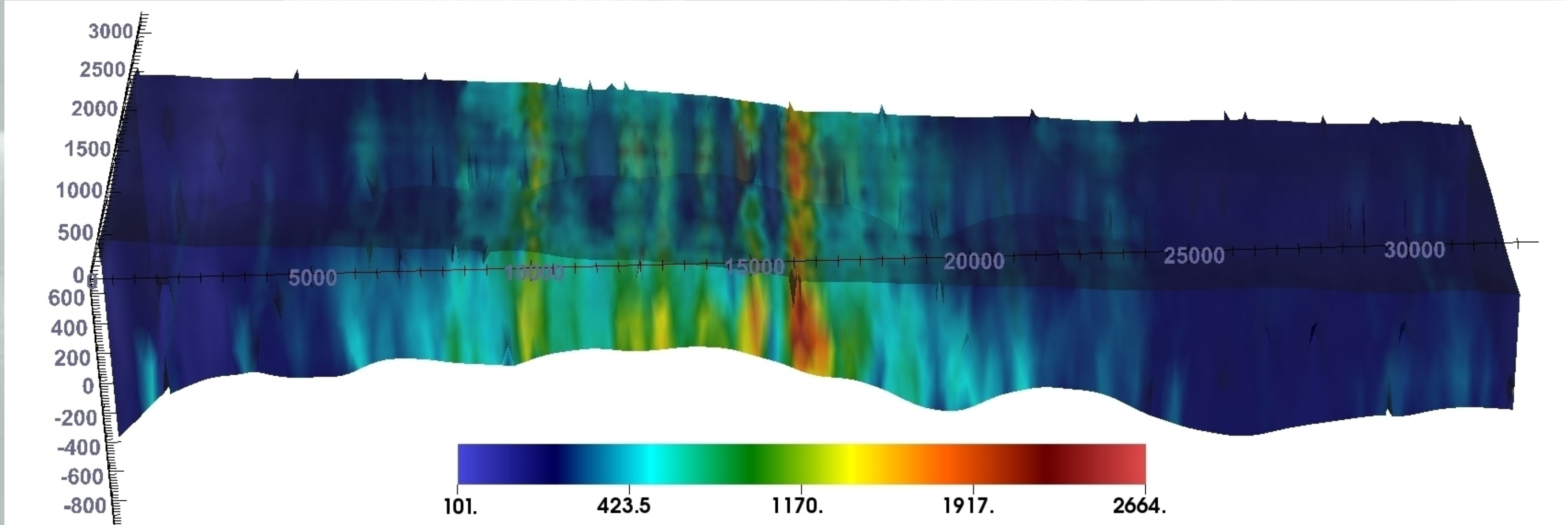

Sensitivity with respect to the consistency

Madec-Martin-Monnier, under preparation

Some perspectives

- Introduce the Bingham threshold into LA algorithm (*a priori* suited for non-differentiable problem)

Glowinski and Wachs, 2011

➡ Herschel-Bulkley constitutive law:

mud flows, avalanches, etc.


Avalanche deposit, Ancey, 2007

- Unsteady adjoint formulation: moving domain (free surface)

shape optimization problem

➡ extra terms due to free-surface movments (*Leibniz formula)*