

Direct and Inverse Variational Modelling of ViscoPlastic Geophysical Flows

APPLICATION TO GLACIOLOGY

Nathan Martin
INSA Toulouse – IMT – Equipe MIP
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CONTENT

- I. Geophysical context
- II. Inverse method
- III. A new finite element solver
- IV. Virtual inverse rheometry
- V. The basal slipperiness
- VI. Conclusions & perspectives

CONTENT

I. Geophysical context

- Cryosphere & sea level rise
- Coastal mass balance
- Ice-flow dynamics
- Ice-flow mechanics

II. Inverse method

III. A new finite element solver

IV. Virtual inverse rheometry

V. The basal slipperiness

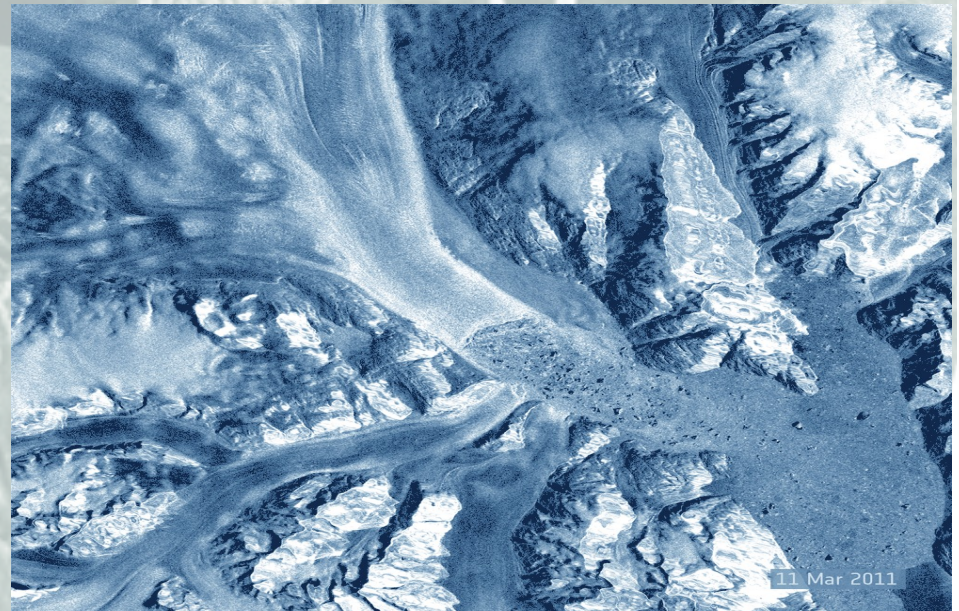
VI. Conclusions & Perspectives

GEOFYSICAL CONTEXT

- Free-surface gravity-driven flow
- High viscosity, Low Reynolds
- Shallow or not...
- Viscoplastic (power-law)



Mount etna eruption: 5/10/08
Robertson & Kerr, JGR, 2012



Kangerdlugssuaq ice streams:
11/03/11, ESA's ERS-2 satellite

Cryosphere and Sea-Level Rise

Land ice

- 75% world's freshwater: → glacier, ice caps, ice-sheet

Mountain glacier

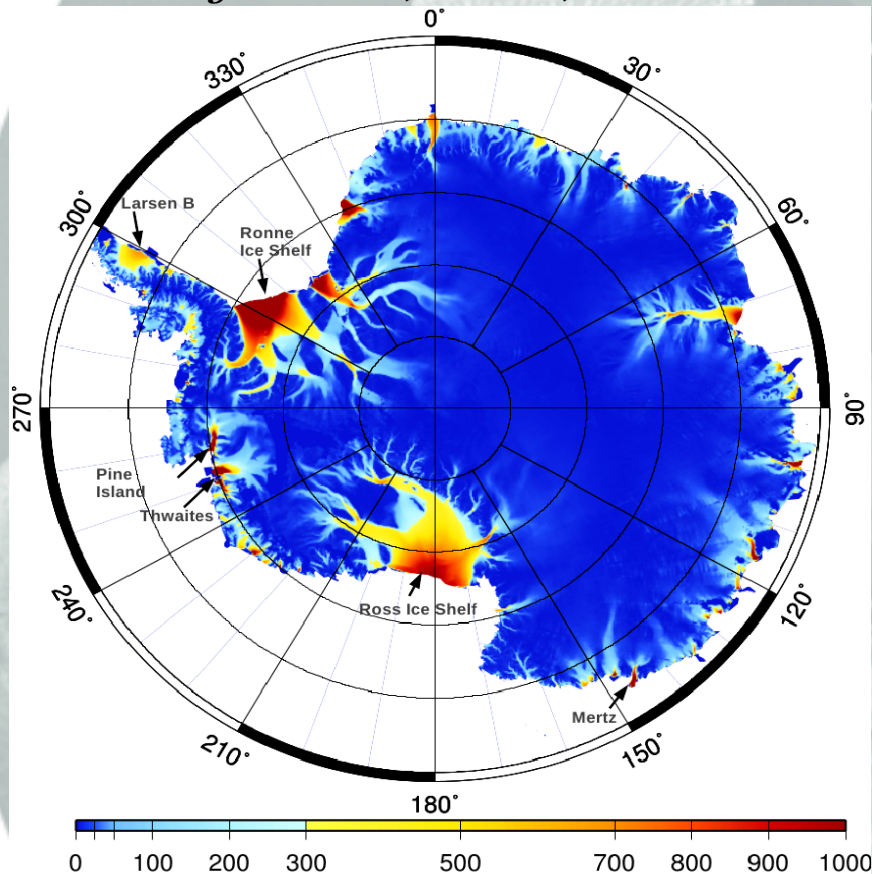
- made of dense snow (precipitations)
- move by deformation (creep) and sliding



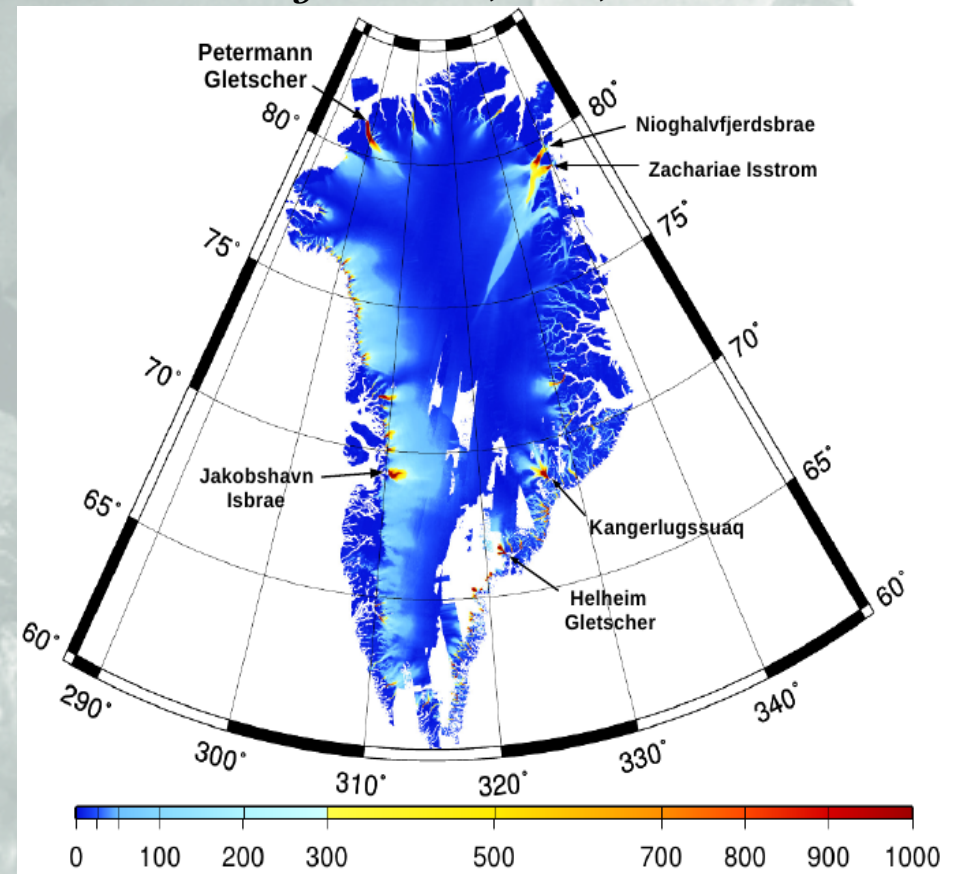
Société suisse de géomorphologie, glacier des Corbassières

Cryosphere and sea level rise

Rignot et al., Science, 2011



Joughin et al., JOG, 2010

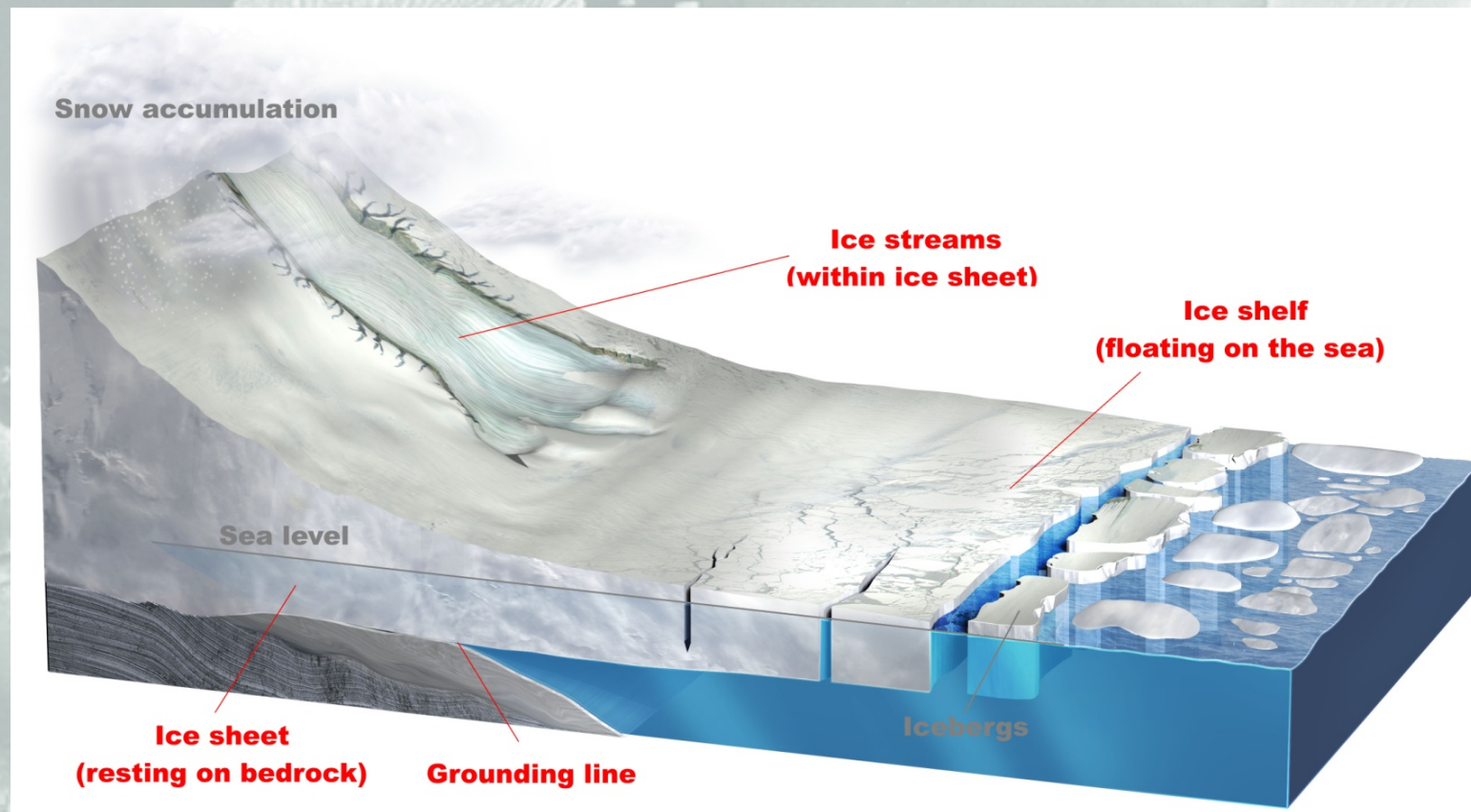


Surface velocities (m/a)

Global warming contribution to SLR: *negative terrestrial ice mass balance*
 Coastal mass loss: Antarctica= 15% of the coast / 90% ice discharge

Coastal mass balance

Mass loss at coastal margin: (hotter) ocean erosion and calving



Bell, Scientific American, 2008, courtesy of Kevin Hand

Ice sheet/stream/shelf: *different basal conditions*

Grounding line: *transition line* from grounded to floating ice

Ice flow dynamics

- Stokes system: $-\operatorname{div}(\underline{\sigma}) = f$, $\nabla \cdot \mathbf{u} = 0$
- Free surface: $\partial_t h + u_x \partial_x h = a + u_z$
- Basal modelling:

Ice/Bedrock
interfacial layer

➔ Sliding law

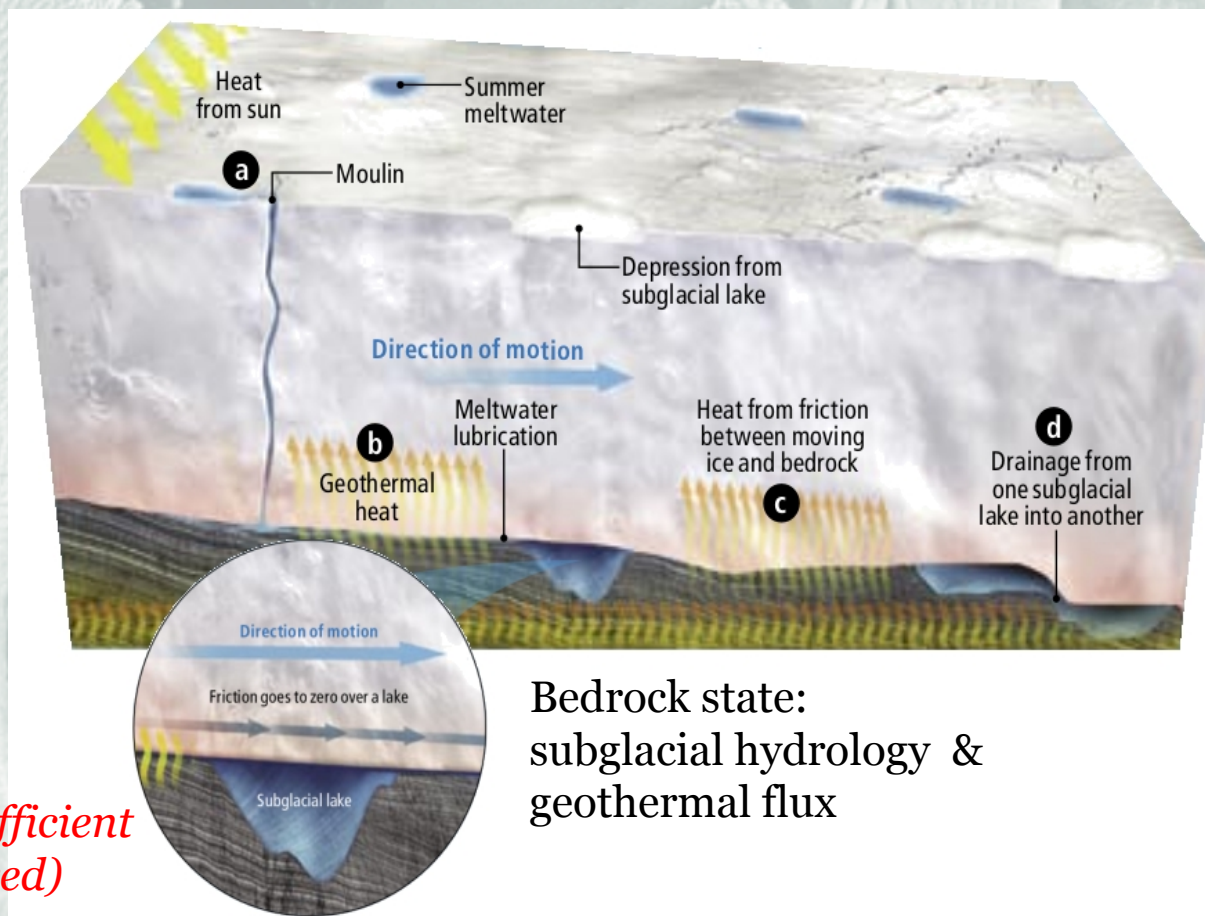
link basal velocity u_b
to basal shearing τ_b

Weertman model

$$|\tau_b|^{m-1} \tau_b = \beta u_b^\tau$$

$$\mathbf{u} \cdot \mathbf{n} = 0$$

*Friction coefficient
(to be inferred)*



Bedrock state:
subglacial hydrology &
geothermal flux

Bell, Scientific American, 2008, courtesy of Kevin Hand

Ice flow mechanics

- Viscous fluid

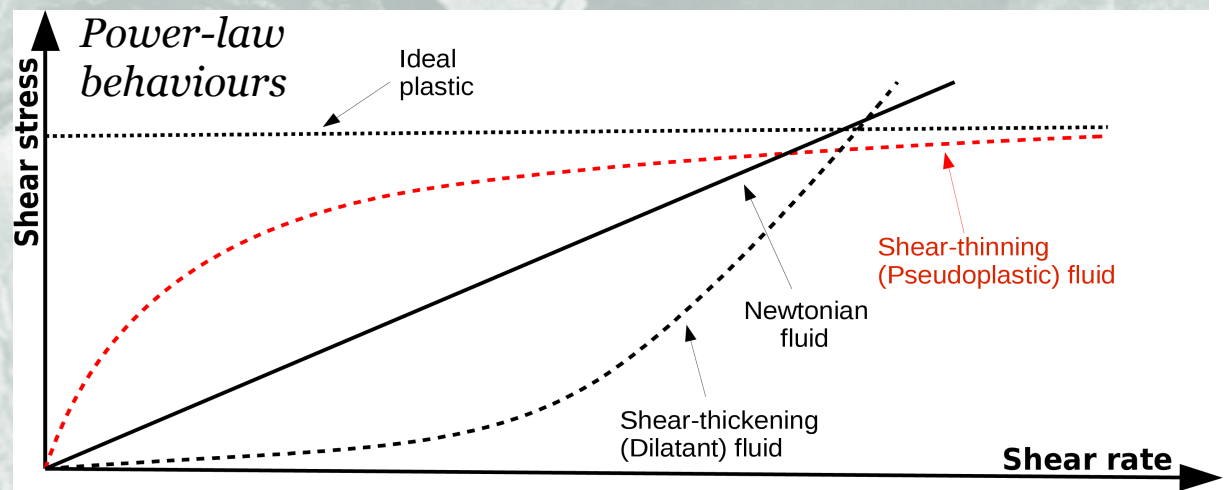
$$\underline{S}(\mathbf{u}) = 2\eta(\mathbf{u})\underline{D}(\mathbf{u}), \quad \underline{D}(\mathbf{u}) = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$$

- Power-law (apparent viscosity) $\eta(\mathbf{u}) = \underset{\substack{\uparrow \\ \text{Consistency + thermal physics} \\ \text{(to be inferred)}}}{\eta_0(T)} \|\underline{D}(\mathbf{u})\|_F^{s-2}$, $\overset{\substack{\text{Power-law exponent} \\ \text{(to be inferred)}}}{s}$
 - $1 < s < 2$: shear-thinning
 - $s > 2$: shear-thickening
 - $s = 2$: Newtonian

Thermal physics at the bottom



State of the bedrock-ice interaction



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- I. Geophysical context
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 - Data in glaciology
 - The adjoint method
 - Variational data assimilation
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INVERSE MODELLING

Modelling aspects

- Non-linear elliptic forward problem
- Adjoint-based approach using algorithmic differentiation
- Multi-physics, uncertain parameters

Observational aspects

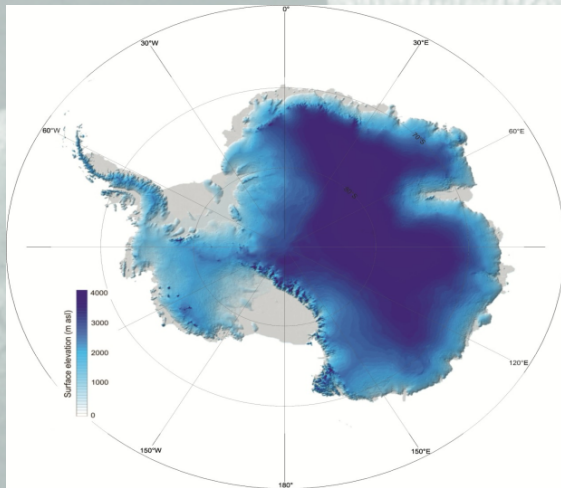
- Difficult measurements: rheology, basal sliding
- Types of observations:
 - In situ: sparse, difficult to perform
 - Remote sensing (airborne, satellite): large scale



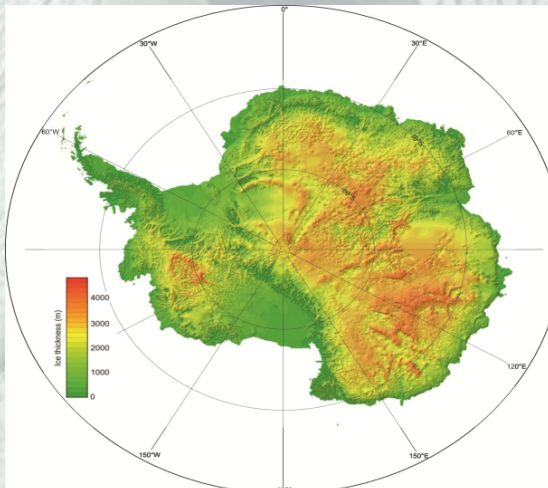
➡ more and more data available

Data in glaciology

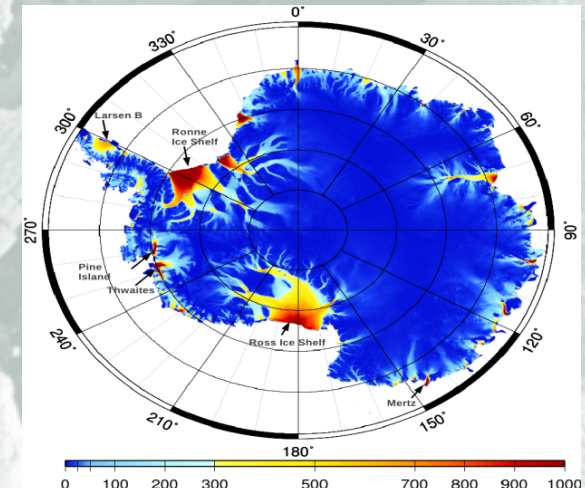
- Types of data & precision
 - *Surface topography*: $\sim 1\text{-}10\text{m}$ (satellite altimetry, stereoscopy, InSAR)
 - *Ice thickness*: $\sim 10\text{-}100\text{m}$ (low frequency airborne/ground radar)
 - *Surface velocities*: $\sim 1\text{-}20\text{m/a}$ (InSAR, offset speckle tracking)



Surface topography, BEDMAP2, 2007



Ice thickness, BEDMAP2, 2007

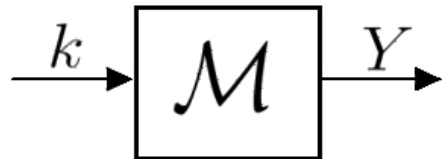


Surface velocities, Rignot et al., 2011

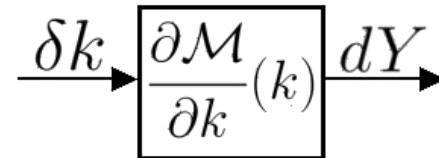
Goal : Infer the characteristics of the flow from indirect observations

The adjoint method

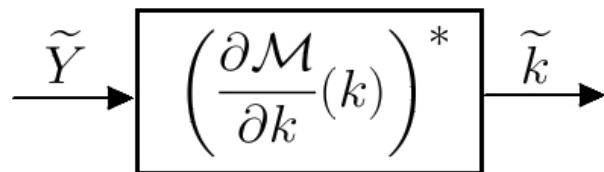
Schematic view of the adjoint model



Direct Model : $Y = \mathcal{M}(k)$



Linear tangent model : $dY = \frac{\partial \mathcal{M}}{\partial k}(k) \cdot \delta k$



Adjoint model : $\tilde{k} = \left(\frac{\partial \mathcal{M}}{\partial k} \right)^* \cdot \tilde{Y}$

where $\left\langle \tilde{Y}, \left(\frac{\partial \mathcal{M}}{\partial k} \right) \cdot \delta k \right\rangle = \left\langle \left(\frac{\partial \mathcal{M}}{\partial k} \right)^* \cdot \tilde{Y}, \delta k \right\rangle$

Courtesy of Honnorat, PhD, 2007

Output of the adjoint code: gradient of the cost function w.r.t. the control parameters, for all directions δk :

$\tilde{k} = \frac{\partial j}{\partial k}(k) \longleftrightarrow$ **Sensitivity** of the model w.r.t its parameterization

Variational data assimilation

**OPTIMAL
CONTROL LOOP**

COST FUNCTION (to be minimized)

$$\min_{\mathbf{k}} j(\mathbf{k}) = \frac{1}{2} \int \|C\mathbf{y}(\mathbf{k}) - \mathbf{y}^{obs}\|_*^2 d\mathbf{x} \\ + \text{regularization terms}$$

**MISFIT BETWEEN
COMPUTATIONS
AND OBSERVATIONS**

Observations (sampling
from an unknown true state)



Input Parameters (to be controlled):

Empirical coefficients (rheological parameters, basal friction,...)

**Flow Model (constraint for the
minimization process):**

- non-linear quasi static Stokes
- Moving free-surface

Output (numerical results):

- Velocity, pressure, deformation
- Extra-response: viscosity, mass balance