INSTITUT NATIONAL POLYTECHNIQUE DE TOULOUSE

MATHS Rappels
Formulaire

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II.1 Fonctions trigonométriques

sinus	cosinus	tangente
$\sin(\pi/2) = 1$	$\cos(\pi/2) = 0$	$\tan(\pi/2) = \infty$
$\sin(\pi/3) = \sqrt{3}/2$	$\cos(\pi/3) = 1/2$	$\tan(\pi/3) = \sqrt{3}$
$\sin(\pi/4) = \sqrt{2}/2$	$\cos(\pi/4) = \sqrt{2}/2$	$\tan(\pi/4) = 1$
$\sin(\pi/6) = 1/2$	$\cos(\pi/6) = \sqrt{3}/2$	$\tan(\pi/6) = \sqrt{3}/3$
$\sin(-a) = -\sin(a)$	$\cos(-a) = \cos(a)$	$\tan(-a) = -\tan(a)$
$\sin(\pi - a) = \sin(a)$	$\cos(\pi - a) = -\cos(a)$	$\tan(\pi - a) = -\tan(a)$
$\sin(\pi + a) = -\sin(a)$	$\cos(\pi + a) = -\cos(a)$	$\tan(\pi + a) = \tan(a)$
$\sin(\pi/2 - a) = \cos(a)$	$\cos(\pi/2 - a) = \sin(a)$	$\tan(\pi/2 - a) = 1/\tan(a)$
$\sin(\pi/2 + a) = -\cos(a)$	$\cos(\pi/2 + a) = -\sin(a)$	$\tan(\pi/2 + a) = -1/\tan(a)$

Formules d'addition

$$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$$
$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$\tan(a+b) = \frac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)}$$
$$\sin(a-b) = \sin(a)\cos(b) - \sin(b)\cos(a)$$
$$\cos(a-b) = (\cos(a)\cos(b) + \sin(a)\sin(b)$$
$$\tan(a-b) = \frac{\tan(a)-\tan(b)}{1+\tan(a)\tan(b)}$$
$$\sin(2a) = 2\sin(a)\cos(a)$$
$$\cos(2a) = \cos^2(a) - \sin^2(a)$$
$$\tan(2a) = \frac{2\tan(a)}{1-\tan^2(a)}$$

transformation somme-produit :

$$\sin(a) + \sin(b) = 2\sin(\frac{a+b}{2})\cos(\frac{a-b}{2})$$
$$\cos(a) + \cos(b) = 2\cos(\frac{a+b}{2})\cos(\frac{a-b}{2})$$
$$\tan(a) + \tan(b) = \frac{\sin((a+b)}{\cos(a)\cos(b)}$$
$$\sin(a) - \sin(b) = 2\cos(\frac{a+b}{2})\sin(\frac{a-b}{2})$$
$$\cos(a) - \cos(b) = -2\sin(\frac{a+b}{2})\sin(\frac{a-b}{2})$$
$$\tan(a) - \tan(b) = \frac{\sin(a-b)}{\cos(a)\cos(b)}$$

Relations usuelles :

$$\sin^2(x) + \cos^2(x) = 1$$
$$\sin^2(x) = \frac{1-\cos(2x)}{2} = \frac{\tan^2(x)}{1+\tan^2(x)}$$
$$\cos^2(x) = \frac{1+\cos(2x)}{2} = \frac{1}{1+\tan^2(x)}$$

transformation : $t = \tan(\frac{a}{2})$

$$\sin(a) = \frac{2t}{1+t^2}$$
$$\cos(a) = \frac{1-t^2}{1+t^2}$$
$$\tan(a) = \frac{2t}{1-t^2}$$

Transformation produit-somme :

$$\sin(a)\sin(b) = \frac{\cos(a-b) - \cos(a+b)}{2}$$
$$\cos(a)\cos(b) = \frac{\cos(a-b) + \cos(a+b)}{2}$$
$$\sin(a)\cos(b) = \frac{\sin(a+b) + \sin(a-b)}{2}$$

II.2 Fonctions hyperboliques

Définitions

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$
$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$
$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$
$$\cosh(x) + \sinh(x) = e^x$$
$$\cosh(x) - \sinh(x) = e^{-x}$$

Formules d'addition

$$\cosh(a+b) = \cosh(a)\cosh(b) + \sinh(b)\sinh(a)$$
$$\sinh(a+b) = \sinh(a)\cosh(b) + \sinh(b)\cosh(a)$$
$$\tanh(a+b) = \frac{\tanh(a) + \tanh(b)}{1 + \tanh(a)\tanh(b)}$$
$$\cosh(a-b) = \cosh(a)\cosh(b) - \sinh(a)\sinh(b)$$
$$\sinh(a-b) = (\sinh(a)\cosh(b) - \cosh(a)\sinh(b)$$
$$\tanh(a-b) = \frac{\tanh(a) - \tanh(b)}{1 - \tanh(a)\tanh(b)}$$
$$\sinh(2a) = 2\sinh(a)\cosh(a) = \frac{2\tanh a}{1 - \tanh^2 a}$$
$$\cosh(2a) = \cosh^2(a) + \sinh^2(a) = 2\cosh^2 a - 1 = 2\sinh^2 a + 1 = \frac{1 + \tanh^2(a)}{1 - \tanh^2(a)}$$
$$\tanh(2a) = \frac{2\tanh(a)}{1 + \tanh^2(a)}$$

transformation : $t = \tanh(\frac{a}{2})$

$$\sinh(a) = \frac{2t}{1 - t^2}$$
$$\cosh(a) = \frac{1 + t^2}{1 - t^2}$$
$$\tanh(a) = \frac{2t}{1 + t^2}$$

Relations usuelles :

$$\cosh(-x) = \cosh(x)$$
$$\sinh(-x) = -\sinh(x)$$
$$\tanh(-x) = -\tanh(x)$$
$$\coth(-x) = -\coth(x)$$
$$\cosh^2(x) - \sinh^2(x) = 1$$
$$\sinh^2(x) = \frac{\tanh^2(x)}{1 - \tanh^2(x)}$$
$$\cosh^2(x) = \frac{1}{1 - \tanh^2(x)}$$
$$\cosh^2(\frac{x}{2}) = \frac{\cosh(x) + 1}{2}$$
$$\sinh^2(\frac{x}{2}) = \frac{\cosh(x) - 1}{2}$$
$$\tanh(\frac{x}{2}) = \frac{\sinh(x)}{\cosh(x) + 1} = \frac{\cosh(x) - 1}{\sinh(x)} = \pm\sqrt{\frac{\cosh(x) - 1}{\cosh(x) + 1}}$$

transformation somme-produit :

$$\sinh(a) + \sinh(b) = 2\sinh(\frac{a+b}{2})\cosh(\frac{a-b}{2})$$
$$\cosh(a) + \cosh(b) = 2\cosh(\frac{a+b}{2})\cosh(\frac{a-b}{2})$$
$$\tanh(a) + \tanh(b) = \frac{\sinh(a+b)}{\cosh(a)\cosh(b)}$$
$$\sinh(a) - \sinh(b) = 2\cosh(\frac{a+b}{2})\sinh(\frac{a-b}{2})$$
$$\cosh(a) - \cosh(b) = 2\sinh(\frac{a+b}{2})\sinh(\frac{a-b}{2})$$
$$\tanh(a) - \tanh(b) = \frac{\sinh(a-b)}{\cosh(a)\cosh(b)}$$

II.3 Développements limités

Table II.1: Développement limités de fonctions usuelles

fonction	Développement limité d'ordre n au voisinage de 0
e^x	$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + x^n \epsilon(x)$
$\sin(x)$	$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^p \frac{x^{2p+1}}{(2p+1)!} + x^{2p+2}\epsilon(x)$
$\cos(x)$	$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + (-1)^p \frac{x^{2p}}{(2p)!} + x^{2p+1}\epsilon(x)$
$\tan(x)$	$\tan(x) = x + \frac{x^3}{3} + 2\frac{x^5}{15} + 17\frac{x^7}{315} + x^8\epsilon(x)$
$sh(x)$	$sh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2p+1}}{(2p+1)!} + x^{2p+2}\epsilon(x)$
$ch(x)$	$ch(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2p}}{(2p)!} + x^{2p+1}\epsilon(x)$
$th(x)$	$th(x) = x - \frac{x^3}{3} + 2\frac{x^5}{15} - 17\frac{x^7}{315} + x^8\epsilon(x)$
$\frac{1}{1+x}$	$\frac{1}{1+x} = 1 - x + x^2 + \cdots + (-1)^n x^n + +x^n\epsilon(x)$
$\frac{1}{1-x}$	$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + +x^n\epsilon(x)$
$\frac{1}{(1+x)^2}$	$\frac{1}{(1+x)^2} = 1 - 2x + 3x^2 + \cdots + (-1)^n(n+1)x^n + x^n\epsilon(x)$
$\sqrt{1+x}$	$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{2.4} + \cdots + (-1)^{n+1}\frac{1.3.5\cdots(2n-3)}{2.4\cdots(2n)}x^n + x^n\epsilon(x)$
$\frac{1}{\sqrt{1+x}}$	$\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{1.3}{2.4}x^2 + \cdots + (-1)^{n+1}\frac{1.3.5\cdots(2n-1)}{2.4\cdots(2n)}x^n + x^n\epsilon(x)$
$(1+x)^\alpha, \alpha \in \mathbb{Q}$	$(1+x)^\alpha = 1 + \frac{\alpha}{1!}x + \frac{\alpha(\alpha-1)}{2!}x^2 + \cdots + \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n + x^n\epsilon(x)$
$\ln(1+x)$	$\ln(1+x) = x - \frac{x^2}{2} + \cdots + (-1)^{n+1}\frac{x^n}{n} + x^n\epsilon(x)$
$\arcsin(x)$	$\arcsin((x) = x + \frac{1}{2.3}x^3 + \cdots + \frac{1.3.5\cdots(2p-1)}{2.4.6\cdots(2p)(2p+1)}x^{2p+1} + x^{2p+2}\epsilon(x)$
$Argsh(x)$	$Argsh(x) = x - \frac{1}{2.3}x^3 + \cdots + (-1)^p\frac{1.3.5\cdots(2p-1)}{2.4.6\cdots(2p)(2p+1)}x^{2p+1} + x^{2p+2}\epsilon(x)$
$\arctan(x)$	$Arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots + (-1)^p\frac{x^{2p+1}}{2p+1} + x^{2p+2}\epsilon(x)$
$Argth(x)$	$Argth(x) = x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{x^{2p+1}}{2p+1} + x^{2p+2}\epsilon(x)$
$\ln(\frac{1+x}{1-x})$	$\ln(\frac{1+x}{1-x}) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{x^{2p+1}}{2p+1}\right) + x^{2p+2}\epsilon(x)$

II.4 Dérivation-intégration

TABLE II.2: Dérivées et primitives de fonctions usuelles

fonction	Dérivée	*Primitive*	*Conditions*
a	0	ax	
x^α	$\alpha x^{\alpha-1}$	$\frac{x^{\alpha+1}}{\alpha+1}$	$\alpha \neq -1, x > 0$
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\ln(x)$	$x \neq 0$
$\frac{1}{a^2+x^2}$	$-\frac{2x}{(a^2+x^2)^2}$	$\frac{1}{a} Arctan\left(\frac{x}{a}\right)$	$a \neq 0$
$\frac{1}{a^2-x^2}$	$\frac{2x}{(a^2-x^2)^2}$	$\frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\|$	$a \neq 0, x \neq \pm a$
$\frac{1}{x^2-a^2}$	$-\frac{2x}{(x^2-a^2)^2}$	$\frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\|$	$a \neq 0, x \neq \pm a$
$\sqrt{x^2+a^2}$	$\frac{x}{\sqrt{x^2+a^2}}$	$\frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln(x+\sqrt{x^2+a^2})$	$a \neq 0$
$\sqrt{x^2-a^2}$	$\frac{x}{\sqrt{x^2-a^2}}$	$\frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left\|x+\sqrt{x^2-a^2}\right\|$	$a \neq 0, \|x\| > \|a\|$
$\sqrt{a^2-x^2}$	$-\frac{x}{\sqrt{a^2-x^2}}$	$\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin(\frac{x}{a})$	$a > 0, \|x\| < a$
$\frac{1}{\sqrt{x^2+a^2}}$	$-\frac{x}{\sqrt{(x^2+a^2)^3}}$	$\ln(x+\sqrt{x^2+a^2})$	$a \neq 0$
$\frac{1}{\sqrt{x^2-a^2}}$	$-\frac{x}{\sqrt{(x^2-a^2)^3}}$	$\ln\left\|x+\sqrt{x^2-a^2}\right\|$	$a \neq 0, \|x\| > \|a\|$
$\frac{1}{\sqrt{a^2-x^2}}$	$\frac{x}{\sqrt{(a^2-x^2)^3}}$	$\arcsin(\frac{x}{a})$	$a \neq 0, \|x\| < a$
e^x	e^x	e^x	
a^x	$a^x \ln(a)$	$\frac{a^x}{\ln(a)}$	$a > 0, a \neq 1$
$\ln(x)$	$\frac{1}{x}$	$x\ln(x) - x$	$x > 0$
$log_a(x)$	$\frac{1}{x\ln(a)}$	$x(log_a(x) - log_a(e))$	$a > 0, a \neq 1, x > 0$
$\sin(x)$	$\cos(x)$	$-\cos(x)$	
$\cos(x)$	$-\sin(x)$	$\sin(x)$	
$tan(x)$	$1 + tan^2(x) = \frac{1}{\cos^2(x)}$	$-\ln\|\cos(x)\|$	$x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$
$cotan(x)$	$-(1 + cotan^2(x)) = -\frac{1}{\sin^2(x)}$	$\ln\|\sin(x)\|$	$x \neq k\pi, k \in \mathbb{Z}$
$\frac{1}{\sin(x)}$	$-\frac{\cos(x)}{\sin^2(x)}$	$\ln\left\|tan(\frac{x}{2})\right\|$	$x \neq k\pi, k \in \mathbb{Z}$
$\frac{1}{\cos(x)}$	$\frac{\sin(x)}{\cos^2(x)}$	$\ln\left\|tan(\frac{x}{2} + \frac{\pi}{4})\right\|$	$x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

$fonction$	Dérivée	$Primitive$	$Conditions$
$\dfrac{1}{\cos^2(x)}$	$-2\tan(x)(1+\tan^2(x))$	$\tan(x)$	$x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$
$\arcsin((x)$	$\dfrac{1}{\sqrt{1-x^2}}$	$x\arcsin(x) + \sqrt{1-x^2}$	$\|x\| < 1$
$\arccos(x)$	$\dfrac{-1}{\sqrt{1-x^2}}$	$x\arccos(x) - \sqrt{1-x^2}$	$\|x\| < 1$
$\arctan(x)$	$\dfrac{1}{1+x^2}$	$x\arctan(x) - \ln(\sqrt{1+x^2})$	
$arccotan(x)$	$\dfrac{-1}{1+x^2}$	$xarccotan(x) + \ln(\sqrt{1+x^2})$	
$sh(x)$	$ch(x)$	$ch(x)$	
$ch(x)$	$sh(x)$	$sh(x)$	
$th(x)$	$1 - th^2(x) = \dfrac{1}{ch^2(x)}$	$\ln(ch(x))$	
$coth(x)$	$1 - coth^2(x) = -\dfrac{1}{sh^2(x)}$	$\ln\|sh(x)\|$	$x \neq 0$
$\dfrac{1}{ch^2(x)}$	$-2\dfrac{th(x)}{ch^2(x)}$	$th(x)$	
$Argsh(x)$	$\dfrac{1}{\sqrt{1+x^2}}$	$xArgsh(x) - \sqrt{1+x^2}$	1
$Argch(x)$	$\dfrac{1}{\sqrt{x^2-1}}$	$xArgch(x) - \sqrt{x^2-1}$	$x > 1$
$Argth(x)$	$\dfrac{1}{1-x^2}$	$xArgth(x) + \ln(\sqrt{1-x^2})$	$\|x\| < 1$
$Argcoth(x)$	$\dfrac{1}{1-x^2}$	$xArgcoth(x) + \ln(\sqrt{x^2-1})$	$\|x\| > 1$

II.5 Binôme de newton

Le binôme de Newton est une formule mathématique donnée par Isaac Newton pour le développement d'une puissance entière quelconque d'un binôme.

$$(x+y)^n = \sum_{k=0}^{n} C_n^k x^{n-k} y^k$$

où les nombres :

$$C_n^k = \frac{n!}{k!(n-k)!}$$

sont appelés coefficients binomiaux.

$$C_{n+1}^{k+1} = C_n^k + C_n^{k+1}$$

$$C_n^k = C_{n-k}^k$$

$$C_n^k = \frac{n}{k} C_{n-1}^{k-1}$$